Magnetic Field Lines :

- **Definition :** The magnetic field lines are a visual and intuitive realization of the magnetic field. It is the path along which an isolated north pole will tend to move, if placed inside a magnetic field.

- **Properties :**

  (i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops.

  (ii) The tangent to the field line at a given point represents the direction of the net magnetic field **B** at the point.

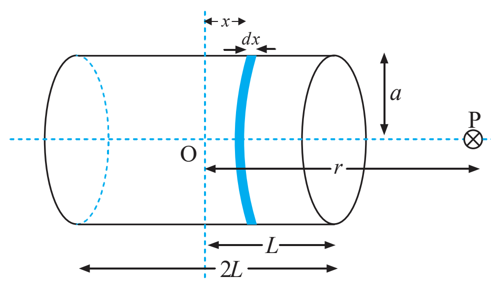
  (iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field **B**.

  (iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

Bar Magnet as an Equivalent Solenoid :

Let us calculate the axial field of a finite solenoid and we will demonstrate that at large distances, this axial field resembles that of a bar magnet.

- Let the length of the solenoid be $2L$, radius a, it consists of n turns per unit length and I be the current in the solenoid.
- Let us evaluate the axial field at a point P, at a distance r from the centre 0 of the solenoid.
- Consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of ndx turns.
- The magnitude of the field at point P due to the circular element is :



Calculation of the axial field of a finite solenoid in order to demonstrate its similarity to that of a bar magnet.

$$dB = \frac{\mu_0 I a^2}{2[(r-x)^2 + a^2]^{3/2}} \times (n.dx)$$

Integrate from $x = -L$ to $x = +L$ for obtaining the magnitude of total field.

$$B = \frac{\mu_0 I a^2 n}{2} \int_{-L}^{+L} \frac{dx}{[(r-x)^2 + a^2]^{3/2}}$$

Considering the far axial field of the solenoid i.e. $r >> a$ and $r >> L$, then denominator is approximated by :

$$[(r-x)^2 + a^2]^{3/2} = r^3 \quad ; \quad B = \frac{\mu_0 I a^2 n}{2r^3}\int_{-L}^{L} dx$$

So, $$B = \frac{\mu_0 I a^2 n}{2r^3}\int_{-L}^{L} dx = \frac{\mu_0 n I}{2}\frac{2La^2}{r^3}$$

The magnitude of the magnetic moment of the solenoid is :

$$m = (\text{Total number of turns}) \times (\text{current}) \times (\text{cross-sectional area}) \Rightarrow m = n(2L)\, I\, (\pi a^2)$$

So, $$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

- This is also the far axial magnetic field of a bar magnet.

  Thus, a bar magnet and a solenoid produce similar magnetic fields.

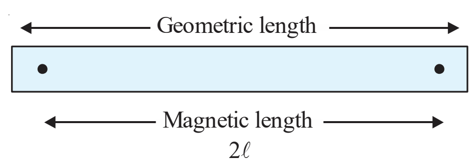
- The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

Magnetic Moment of a Bar Magnet

Magnetic poles are slightly inside the ends of a bar magnet and distance between the magnetic poles is called magnetic length of a bar magnet. It is written as $2l$.

Observed relation between geographic length and magnetic length of a bar magnet is given below.

$$\frac{\text{Magnetic length}}{\text{Geometric length}} = 0.84$$



Magnetic moment of a bar magnet is defined as

$$M = m \times 2l$$
$$= 2ml$$

where $2l$ is the magnetic length and m is the pole strength. It is a vector quantity and its direction is from south pole to north pole. SI unit of magnetic moment is $A\text{-}m^2$.

Magnetic Dipole in a Uniform Magnetic field :

To determine the magnitude of $\boldsymbol{B}$ accurately, a small compass needle of known magnetic moment $\boldsymbol{m}$ and moment of inertia $\boldsymbol{I}$ is allowed to oscillate in the magnetic field.

The torque on the needle is $\vec{\tau} = \vec{m} \times \vec{B}$

In magnitude, $\tau = mB\sin\theta$

Here τ is restoring torque and θ is the angle between $\vec{m}$ and $\vec{B}$.

Therefore, $$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

Negative sign with $mB\sin\theta$ implies that restoring torque is in opposition to deflecting torque. For small values of θ, $\sin\theta \approx \theta$.

So, $$I\frac{d^2\theta}{dt^2} \approx -mB\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} = \frac{-mB\theta}{I}$$

This represents a Simple Harmonic Motion

with $\omega^2 = \frac{mB}{I}$, ($\omega \rightarrow$ angular frequency)

and $T = 2\pi\sqrt{\frac{I}{mB}}$, ($T \rightarrow$ time period) or $B = \frac{4\pi^2 I}{mT^2}$

Magnetic Potential Energy (U_m) :

The magnetic potential energy U_m is given by

$$U_m = \int \tau(\theta) \, . \, d\theta$$

$$= \int mB \sin\theta \; d\theta = -mB\cos\theta = -\vec{m} \cdot \vec{B}$$

Taking the constant of integration to be zero means fixing the zero of potential energy at $\theta = 90°$ i.e. when the needle is perpendicular to the field.

- Potential energy is minimum at $\theta = 0°$, i.e. $U_m = -mB$ (most stable position)
- Potential energy is maximum at $\theta = 180°$, i.e. $U_m = +mB$ (most unstable position).

The Dipole Analogy		
	Electrostatics	**Magnetism**
	$1/\varepsilon_0$	μ_0
Dipole moment	$\vec{p}$	$\vec{m}$
Equatorial Field for a short dipole	$-\vec{p}/4\pi\varepsilon_0 r^3$	$-\mu_0 \vec{m}/4\pi r^3$
Axial Field for a short dipole	$2\vec{p}/4\pi\varepsilon_0 r^3$	$\mu_0 2\vec{m}/4\pi r^3$
External Field : Torque	$\vec{p} \times \vec{E}$	$\vec{m} \times \vec{B}$
External Field : Energy	$-\vec{p} \, . \, \vec{E}$	$-\vec{m} \, . \, \vec{B}$

Gauss's Law for Magnetism :

The net magnetic flux through any closed surface is zero.

$$\oint \vec{B} \, . \, d\vec{s} = 0$$

Helps to conclude that :

(i) The number of field lines entering a surface equals those leaving the surface.

(ii) Isolated magnetic poles (monopoles) do not exist.

Illustration - 26 *A closely wound solenoid of 2000 turns and area of cross-section $1.6\times10^{-4}\ m^2$, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.*

(A) *What is the magnetic moment associated with the solenoid?*

(B) *What are the force and torque on the solenoid, if a uniform horizontal magnetic field of $7.5\times10^{-2}T$ is set up at an angle of 30° with the axis of the solenoid ?*

SOLUTION :

Given, number of turns $n = 2000$

Area of cross-section $A = 1.6\times10^{-4}\,\text{m}^2$

Current $I = 4A$

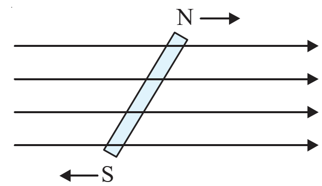
(A) Magnetic moment associated with solenoid

$$M = NIA = 2000\times4\times1.6\times10^{-4} = 1.28\ \text{J/T}$$

(B) The force (net) on the solenoid is zero, because two equal and opposite forces (on each of its poles) are acting. But their lines of action are parallel, so they form a couple. Thus a torque is applied on it.

Torque on the solenoid $\tau = MB\sin\theta$

(Given $\theta = 30°$)



$= 1.28\times7.5\times10^{-2}\ \sin 30°$

$= 1.28\times7.5\times10^{-2}\times\dfrac{1}{2}$

$= 4.8\times10^{-2}\ \text{N-m}$

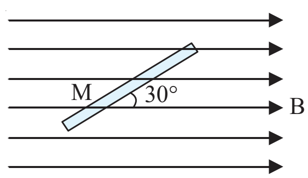
Illustration - 27 *A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} Nm. What is the magnitude of magnetic moment of the magnet?*

SOLUTION :

Given, uniform magnetic field $B = 0.25$ T

The magnitude of torque $\tau = 4.5\times10^{-2}\ Nm$

Angle between magnetic moment and magnetic field $\theta = 30°$.



Torque experienced on a magnet placed in external magnetic field :

$$\tau = M \times B$$

$\tau = MB\sin\theta \qquad (\because A\times B = AB\sin\theta)$

$4.5\times10^{-2} = M\times0.25\times\sin30°$

$$M = \frac{4.5\times10^{-2}}{0.25\times\sin30°}$$

$$= \frac{4.5\times10^{-2}\times2}{0.25\times1} \qquad \left(\because \sin 30° = \frac{1}{2}\right)$$

$= 0.36$ J/T

Thus, the magnitude of magnetic moment of the magnet is 0.36 J/T.

Illustration - 28 *A short bar magnet of magnetic moment m = 0.32 J/T is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (A) stable and (B) unstable equilibrium? What is the potential energy of the magnet in each case ?*

SOLUTION :

Given, magnetic moment of bar magnet $m = 0.32$ J/T

The magnitude of magnetic field $B = 0.15$ T

(A) For stable equilibrium, the angle between magnetic moment (m) and magnetic field **(B)** is $\theta = 0°$

($\because$ In this position, it will be in a direction parallel to magnetic field, so no torque will act on it.)

$\because$ The potential energy of the magnet

$$U = -m.B$$

$$= -mB\cos\theta \quad (\because A.B = AB\cos\theta)$$

$$= -0.32 \times 0.15\cos 0° = -4.8 \times 10^{-2}\,\text{J}$$

Thus, for the stable equilibrium the potential energy is -4.8×10^{-2} J

(B) For the unstable equilibrium, the angle between the magnetic moment and magnetic field is $180°$. ($\because$ In this position it will be in a direction opposite to magnetic field, so maximum torque will act on it when it is displaced slightly)

$$\theta = 180°$$

Potential energy of the magnet

$$U = -mB\cos 180°$$

$$= -0.32 \times 0.15(-1) = 4.8 \times 10^{-2}\,\text{J}$$

Thus, for the unstable equilibrium the potential energy is 4.8×10^{-2} J

Illustration - 29 *A bar magnet of magnetic moment 1.5 J/T lies aligned with the direction of a uniform magnetic field of 0.22T.*

(A) *What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment* : (i) *normal to the field direction* (ii) *opposite to the field direction*?

(B) *What is the torque on the magnet in case* (i) *and* (ii) ?

SOLUTION :

Given, magnetic moment of bar magnet, M = 1.5 J/T

Uniform magnetic field, B = 0.22T

(A) (i) Angle $\theta_1 = 0°$

($\because$ The magnet lies aligned in the direction of field)

and $\theta_2 = 90°$

($\because$ The magnet is to be aligned normal to the field direction)

Work done in rotating the magnet from angle θ_1 to angle θ_2

$$W = -MB(\cos\theta_2 - \cos_1)$$

$$= -1.5 \times 0.22(\cos 90° - \cos 0°) = 0.33\,\text{J}$$

(ii) Angle $\theta_1 = 0°$ and $\theta_2 = 180°$

($\because$ Magnet is to be aligned opposite to the direction of field)

Work done $= -MB(\cos\theta_2 - \cos\theta_1)$

$$= -1.5 \times 0.22(\cos 180° - \cos 0°) = 0.66\,\text{J}$$

(B) Using the formula of torque $\tau = MB\sin\theta$

(i) $\theta = 90°$ (when magnetic moment normal to the field)

$$\tau = 1.5 \times 0.22\sin 90° = 0.33\ \text{N-m}$$

(ii) $\theta = 180°$ (when magnetic moment opposite to the field)

$$\tau = 1.5 \times 0.22\sin 180° = 0$$

Illustration - 30 *A magnetic dipole is under the influence of two magnetic fields. The angle between the two field directions is 60° and one of the fields has a magnitude of 1.2×10^{-2}T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?*

SOLUTION :

Here $B_1 = 1.2 \times 10^{-2} T$

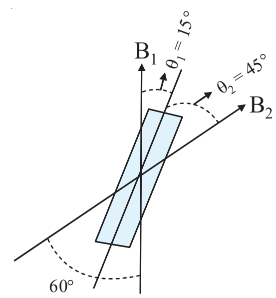
Inclination of dipole with B_1 is $\theta_1 = 15°$

Therefore, inclination of dipole with B_2 is

$$\theta_2 = 60° - 15° = 45°.$$

As the dipole is in equilibrium, therefore the torque on the dipole due to the two fields are equal and opposite. If M is magnetic dipole moment of the dipole, then

$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$$



or $$B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{1.2 \times 10^{-2} \times \sin 15°}{\sin 45°}$$

$$= \frac{1.2 \times 10^{-2} \times 0.2588}{0.707} = 4.39 \times 10^{-3} T$$

Illustration - 31 *A bar magnet with poles* 25 *cm apart and pole-strength* 14.4 *A-m rests with its centre on a frictionless pivot. It is held in equilibrium at* 60° *to a uniform magnetic field of induction* 0.25 *T by applying a force F at right angles to its axis,* 10 *cm from its pivot. Calculate F. What will happen if the force is removed ?*

SOLUTION :

The situation is shown in figure. In equilibrium the torque of M due to B is balanced by torque due to F,

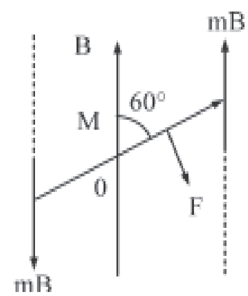
i.e., $$\vec{M} \times \vec{B} = \vec{r} \times \vec{F}$$

$$MB \sin\theta = Fr \sin 90°$$

or $$F = \frac{(m \times 2l)\ B \sin\theta}{r}$$

(as $M = m \times 2l$) ; So substituting the given data,

$$F = \frac{14.4 \times (25 \times 10^{-2}) \times 0.25(\sqrt{3}/2)}{10 \times 10^{-2}} = 7.8\,N$$



If the force $\vec{F}$ is removed, the torque $\vec{M} \times \vec{B}$ will become unbalanced and under its action the magnet will execute oscillatory motion about the direction of B on its pivot O which will not be simple harmonic as initial angular displacement is significant.

Illustration - 32 *Two bar magnets of the same length and breadth but having magnetic moments M and 2M are joined sideways with like poles together and suspended by a string. The time of oscillation of this assembly in a magnetic field of strength B is* 3 *sec. What will be the period of oscillation, if the polarity of one of the magnets is changed and the combination is again made to oscillate in the same field ?*

SOLUTION :

As magnetic moment is a vector, so when magnents are joined with like poles together,

$$M_1 = M + 2M = 3M,$$

So $$T = 2\pi\sqrt{\frac{(I_1 + I_2)}{3MB}} \quad .....(i)$$

When the polarity of one of the magnets is reversed,

$$M_2 = 2M - M = M;$$

So $$T' = 2\pi\sqrt{\frac{(I_1 + I_2)}{MB}} \quad .....(ii)$$

Dividing Equation (ii) by (i)

$$\frac{T'}{T} = \sqrt{3}, i.e., T' = (\sqrt{3})\ T = 3\sqrt{3} \text{ sec}$$

Illustration - 33 *A short bat magnet has magnetic moment of 0.48J/T. Find the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (A) the axis, (B) the equatorial lines (normal bisector) of the magnet.*

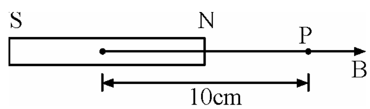
SOLUTION :

Given, magnetic moment of bar magnet $M = 0.48\ J/T$

Distance from the centre of magnet $d = 10\ cm = 0.1\ m$

(A) When the point lies on the axial line.

Magnetic field at point P



$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} = \frac{10^{-7} \times 2 \times 0.48}{(0.1)^3} = 0.96 \times 10^{-4}\,T$$

The direction of magnetic field is along the direction of magnetic moment. We know that the direction of magnetic moment is from S to N pole. Thus, the direction of magnetic field is from S to N pole of the magnet.

(B) Use the formula of magnetic field due to a short bar magnet on its equatorial line.

$\therefore$ Magnetic field at point P

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3} = 10^{-7} \times \frac{0.48}{(0.1)^3} = 0.48 \times 10^{-4}\,T$$

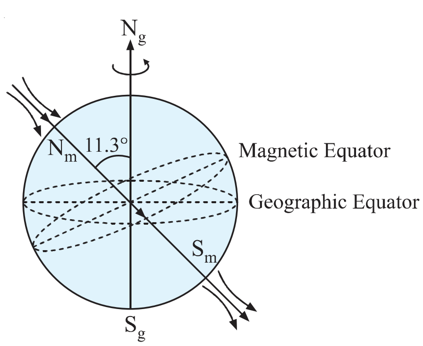
The direction of magnetic field on equatorial line is opposite to the direction of magnetic moment. So, the direction of magnetic field is from N to S pole of the magnet.

The Earth's Magnetism : The magnetic field of Earth is now thought to arise due to electric currents produced by convective motion of metallic fluids (consisting mostly of molten iron and nickel) in the outer core of the earth. This is known as the dynamo effect.

The strength of earth's magnetic field varies from place to place on the earth's surfae ; its value being of the order of 10^{-5} T.

Some facts About Earth's Magnetic field :

- The magnetic field lines of the earth resembles that of a (hypothetical) magnetic dipole located at the centre of the earth.
- The axis of the dipole is tilted by approximately 11.3° with respect to the axis of rotation of the earth.
- The magnetic poles are located where the magnetic field lines due to the dipole enter or leave the earth.
- The pole near the geographic north pole of the earth is called the north magnetic pole (N_m). Likewise, the pole near the geographic south pole is called the south magnetic pole (S_m).



The earth as a giant magnetic dipole

There is some confusion in the nomenclature of the poles. If one looks at the magnetic field lines of the earth, one sees that unlike in the case of a bar magnetic, the field lines go into the earth at the north magnetic pole (N_m) and come out from the south magnetic pole (S_m). The convention arose because the magnetic north was the direction to which the north pole of a magnetic needle pointed ; the north pole of a magnet was so named as it was the north seeking pole. Thus, in reality, the north magnetic pole behaves like the south pole of a bar magnet inside the earth and vice versa.

Geographic Meridian :

The vertical plane containing the longitude circle (determining the geographic north-south direction) and the axis of rotation of the earth is called the geographic meridian.

Magnetic Meridian :

It is defined as the vertical plane which passes through the imaginary line joining the magnetic north and the south poles.

Elements of the Earth's Magnetic Field :

To describe the magnetic field of the earth at a point on its surface, we need to specify three quantities, viz., the declination, the angle of dip or the inclination and the horizontal component of the earth's field. These are known as the elements of the earth's magnetic field.

Elements of Earth's Magnetism

- **Magnetic Declination**
  It is defined as the angle between the geographic and magnetic meridian.
- **Magnetic inclination/dip**
  It is the angle which the earth's total magnetic field makes with the surface of the earth
- **Horizontal Component**
  It is the component of the total intensity of earth's magnetic field in the horizontal direction

Representing the Vertical component by Z_E,
Horizontal component by H_E,

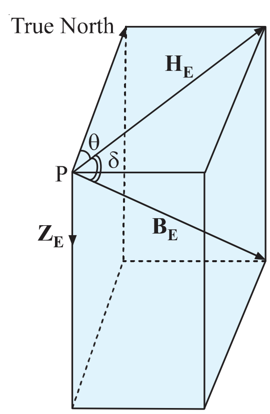
And δ is the angle of dip, we have

$$Z_E = B_E \sin\delta$$

$$H_E = B_E \cos\delta$$

$\Rightarrow$ $$\tan\delta = \frac{Z_E}{H_E}$$

and $$B_E = \sqrt{Z_E^2 + H_E^2}$$



The earth's magnetic field, B_E has its horizontal and vertical components, $\mathbf{H_E}$ and $\mathbf{Z_E}$. Also shown are the declination, θ and the inclination or angle of dip, δ. A magnetic needle freely suspended by a thread attached to its centre will align along B_E with its north pole pointing in the direction of B_E. A compass needle free to rotate in horizontal plane will align along H_E with its north pole in the direction of H_E.

- Angle of dip at poles = 90°
- Angle of dip at equator = 0°
- The declination is greater at higher latitudes and smaller near the equator.

More to Know :

In most of northern hemispheres ; the north pole of the dip needle tilts downwards. Likewise in most of southern hemispheres, south pole of dip needle tilts downwards.

(Think as dip needle is outside the surface of the earth and inner surface of earth behaves as a magnet. In northern hemisphere, Earth behaves as Magnetic South pole and North pole of dip tilts downwards. Similarly in southern hemisphere, Earth surface behaves as magnetic north pole, so south pole of dip needle tilts downwards).

Illustration - 34 *A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at plane is known to be 0.35G. Determine the magnitude of the earth's magnetic field at the place.*

SOLUTION :

Given, angle of dip $\delta = 22°$

Horizontal component of the earth's magnetic field H = 0.35G

Let the magnitude of the earth's magnetic field at the place be B.

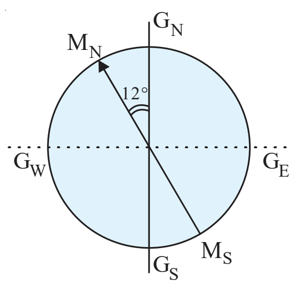
Using the formula, $H = B \cos \delta$

or $$B = \frac{H}{\cos\delta} = \frac{0.35}{\cos 22°} = \frac{0.35}{0.9272} = 0.38G$$

Thus, the value of the earth's magnetic field at that place is 0.38*G*.

Illustration - 35 *At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16G. Specify the direction and magnitude of the earth's field at the location.*

SOLUTION :



Given, angle of declination,

$\theta = 12°$ west

Angle of dip, $\delta = 60°$

Horizontal component of earth's magnetic field

$H = 0.16\,G$

Let the magnitude of earth's magnetic field at that place be B.

Using the formula, $H = B \cos \delta$

Or $$B = \frac{H}{\cos\delta} = \frac{0.16}{\cos 60°} = \frac{0.16 \times 2}{1}$$

$$= 0.32G = 0.32 \times 10^{-4} T$$

The earth's magnetic field lies in a vertical plane 12° west of geographical meridian at angle 60° above the horizontal.

Illustration - 36 *A magnet is suspended so as to swing horizontally. It makes 50 vibrations/min at a place where dip is 30°, and 40 vibrations/min where dip is 45°. Compare the earth's total fields at the two places.*

SOLUTION :

$v \; \alpha \; \sqrt{B_H}$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{B_1 Cos\theta_1}{B_2 Cos\theta_2}}$$

i.e. $$\frac{50}{40} = \sqrt{\frac{B_1}{B_2} \times \frac{Cos30°}{Cos45°}}$$

$$\Rightarrow \quad \frac{25}{16} = \frac{B_1}{B_2} \times \frac{\sqrt{3}}{\sqrt{2}} \quad \text{or} \quad \frac{B_1}{B_2} = \frac{25}{8\sqrt{6}}$$

Illustration - 37 *Considering the earth as a short magnet with its centre coinciding with the centre of earth, show that the angle of dip ϕ is related to magnetic latitude λ through the relation $\tan\phi = 2\tan\lambda$.*

SOLUTION :

Considering the situation for dipole, at position (r, θ) we have

$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \text{ and } B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

and as $\tan\phi = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta}$

So, $\tan\phi = -2\cot(90° + \lambda)$

i.e., $\tan\phi = 2\tan\lambda$

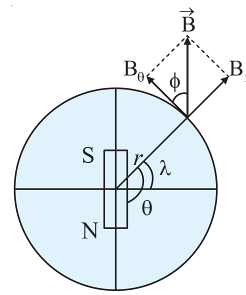


Illustration - 38 *A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null-point (i.e., 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field)*

SOLUTION :

Distance of the null point from the centre of magnet

$$d = 14\,cm = 0.14\,m$$

The earth's magnetic field where the angle of dip is zero, is the horizontal component of earth's magnetic field

i.e., $H = 0.36\,G$

Initially, the null points are on the axis of the magnet. We use the formula of magnetic field on axial line (consider that the magnet is short in length).

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2m}{d^3}$$

This magnetic field is equal to the horizontal component of earth's magnetic field.

i.e., $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2m}{d^3} = H$ ....(i)

On the equatorial line of magnet at same distance **(D)** magnetic field due to the magnet

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{m}{d^3} = \frac{B_1}{2} = \frac{H}{2} \quad ....(ii)$$

The direction of magnetic field on equatorial line at this point (as given in question)

$$B = B_2 + H = \frac{H}{2} + H = \frac{3}{2}H = \frac{3}{2}\times 36 = 0.54\,G$$

The direction of magnetic field is in the direction of earth's field.

Illustration - 39 *A short bar magnet of magnetic moment $5.25\times10^{-2}\,J/T$ is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field on (A) its normal bisector and (B) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distance involved.*

SOLUTION :

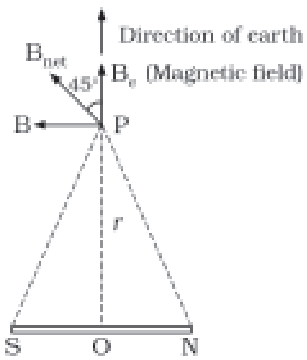
Given, magnetic moment $m = 5.25 \times 10^{-2} J/T$

Let the resultant magnetic field be B_{net}. It makes an angle of $45°$ with B_e.

$\therefore \quad B_e = 0.42G = 0.42 \times 10^{-4} T$

(A) At normal bisector

Let r be the distance between axial line and point P.



The magnetic field at point P, due to a short magnet

$$B = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \quad \ldots \text{(i)}$$

The direction of B is along PA, i.e., along N pole to S pole.

According to vector analysis,

$$\tan 45° = \frac{B}{B_e}$$

$$1 = \frac{B}{B_e} \quad \text{or} \quad B = B_e$$

$$0.42 \times 10^{-4} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

$$0.42 \times 10^{-4} = \frac{10^{-7} \times 5.25 \times 10^{-2}}{r^3}$$

$$r^3 = \frac{5.25 \times 10^{-9}}{0.42 \times 10^{-4}} = 12.5 \times 10^{-5}$$

$r = 0.05\, m$

$r = 5$ cm

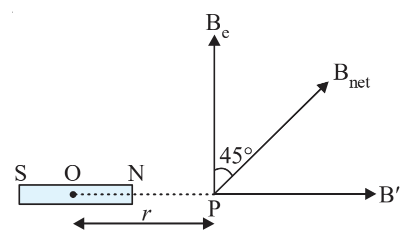
(B) When point lies on axial line

Let the resultant magnetic field B_{net} make an angle of $45°$ with B_e.

The magnetic field on the axial line of the magnet at a distance of r from the centre of magnet

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} \qquad (S \text{ to } N)$$

Direction of magnetic field is from S to N



According to vector analysis,

$$\tan 45° = \frac{B'}{B_e}$$

$$1 = \frac{B'}{B_e} \quad \text{or} \quad B_e = B'$$

$$0.42 \times 10^{-4} = \frac{\mu_0}{4\pi} \times \frac{2m}{r^3}$$

or $$0.42 \times 10^{-4} = \frac{10^{-7} \times 2 \times 5.25 \times 10^{-2}}{r^3}$$

$$r^3 = \frac{10^{-9} \times 2 \times 5.25}{0.42 \times 10^{-4}} = 2.5 \times 10^{-5}$$

$r = 0.063\, m$ or $6.3\, cm$

Magnetisation & Magnetic Intensity :

Definitions :

(i) Magnetisation ($\vec{M}$) : Magnetisation of a sample is equal to its net magnetic moment per unit volume. [A vector quantity measured in units A. m.]

$$\vec{M} = \frac{\vec{m}_{net}}{Vol.}$$

(ii) Magnetic intensity $(\vec{H})$: It represents the degree to which a magnetic material can be magnetized by a magnetic field.

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \qquad (\vec{B} \to \text{Total magnetic field}), \qquad \Rightarrow \qquad \vec{B} = \mu_0\left(\vec{M} + \vec{H}\right)$$

(iii) Magnetic Susceptibility (χ) : It is a measure of how a magnetic material responds to an external field. It is defined as the ratio of intensity of magnetization (M) induced in the material to the magnetic intensity (H).

It is small and +ve for paramagnetic materials and small and negative for diamagnetic materials. In latter case, M and H are opposite in direction. For ferromagnetic materials, χ is large and positive.

$$\chi = \frac{M}{H}$$

- Consider a long solenoid of n-turns per unit length and carrying current I.

  The magnetic field in the interior of the solenoid is given by :

  $$B_0 = \mu_0 I n$$

  If the interior of the solenoid is filled with a material with non-zero magnetization, the net B in the interior of solenoid is given by :

  $$B = B_0 + B_m$$

  Where B_0 : field due to current in solenoid and expressed as

  $$B_0 = \mu_0 In = \mu_0 H \quad (\text{For a solenoid } H = nI)$$

  B_m : field contributed by material core and proportional to M of the material and expressed as

  $$B_m = \mu_0 M$$

  $$B = \mu_0 In + \mu_0 M$$

  $$\Rightarrow \quad B = \mu_0 H + \mu_0 M$$

  $$\Rightarrow \quad B = \mu_0 (H + M)$$

  We have partitioned the contribution to the total magnetic field inside the sample into 2 parts; one due to external factors such as current in solenoid. This is represented by H. The other due to specific nature of magnetic material, namely M. The latter quantity can be influenced by external factors. This influence can be mathematically expressed as $M = \chi H$

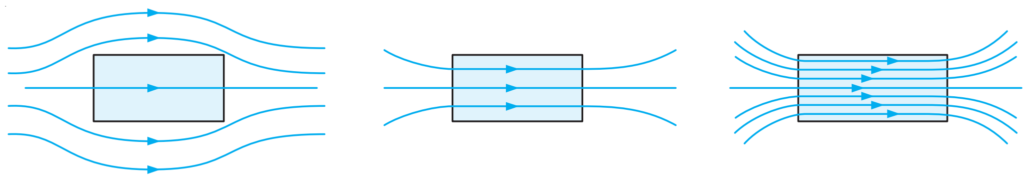
  $$\Rightarrow \quad B = \mu_0 (H + \chi H) \qquad \Rightarrow \quad B = \mu_0 H(1 + \chi)$$

  $$\Rightarrow \quad B = \mu_0 H \mu_r \qquad (1 + \chi = \mu_r \to \text{relatively permeability})$$

  $$\Rightarrow \quad B = \mu H \qquad (\mu = \mu_0 \mu_r)$$

- The three quantities are interrelated and only one of them is independent. Given one, other two may be easily determined.

Classification of Materials on the basis of Magnetic Properties



Diamagnetism	Paramagnetism	Ferromagnetism
The individual atoms (or ions or molecules) do not possess a permanent dipole moment of their own.	The individual atoms (or ions molecules) possess a permanent dipole moment of their own. On account of ceaseless random thermal motion of the atoms, no net magnetisation is seen.	The individual atoms (or ions or molecules) possess a dipole moment. They interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. These domains are randomly oriented and there is no bulk magnetismation.
Application of an external magnetic field $\vec{B}$ induces in each atom, a small dipole moment proportional to B but pointing in the opposite direction. (When $\vec{B}$ is applied electron having orbital magnetic moment in same direction slow down and those in opposite direction speed up because of the Induced current, according to lenz laws. So net magnetic moment is opposite to $\vec{B}$.)	In the presence of an external field $\vec{B}$ which is strong enough, and at low temperatures, the individual atomic dipole moments can be made to align and point in the same direction as $\vec{B}$.	When we apply an external magnetic field B, the domains orient themselves in the direction of B and simultaneously the domain oriented in the direction of B grows in size.
Examples : Bismuth, copper, lead, silicon, water	***Examples :*** Aluminium, sodium, calcium	***Examples :*** Iron, nickel, cobalt, gadolinium.
	Properties	
The field lines are repelled or expelled and the field inside is reduced.	The field lines get concentrated inside the material and the field inside is enhanced.	The field lines are highly concentrated inside the materials.
When placed in a non-uniform magnetic field, the bar will tend to move from stronger to the weaker part of the field. i.e. weakly repelled.	When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong i.e. they get weakly attracted to Magnet.	When placed in non-uniform magnetic field, the bar will tend to move from a region of field weak field to strong field. i.e. Strongly attracted.
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 \le \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
Susceptibility is independent of temperature	Susceptibility depends on temperature $\chi = \frac{c\mu_0}{T}$ [Curie law]	The ferromagnetic property depends on temp. The χ above curie temperature (T_c) is given by : $\chi = \frac{c}{T - T_0}$

Note : **(i)** Superconductors : These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism.

The field lines are completely expelled !

$$\chi = -1 \text{ and } \mu_r = 0$$

(ii) Meissner effect : The phenomenon of perfect diamagnetism in superconductors.

(iii) Magnetisation of a paramagnetic material is directly proportional to the applied field and inversely proportional to the absolute temperature T.

$$M = C\frac{B_0}{T} \text{ (C} \rightarrow \text{Curie's constant)} \quad \text{or} \quad \boxed{\chi = C\frac{\mu_0}{T}} \rightarrow \text{Curie's law} \begin{Bmatrix} \because M = \chi H \\ B_0 = \mu_0 H \end{Bmatrix}$$

As the field is increased or the temperature is lowered, the magnetization increases until it reaches the saturation value M_s, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie's Law is no longer valid.

(iv) At high temperature, a ferromagnet becomes a paramagnet. The domain structure disintegrates with temperature. The temperature of transition from ferromagnetism to paramagnetism is called the CURIE TEMPERATURE (T_c.).

(v) Hard ferromagnets : In some ferromagnetic materials, the magnetization persists even when the external field is removed. Such materials are called hard magnetic materials or hard ferromagnets.

Example **:** Alnico, Iron, Lodestone.

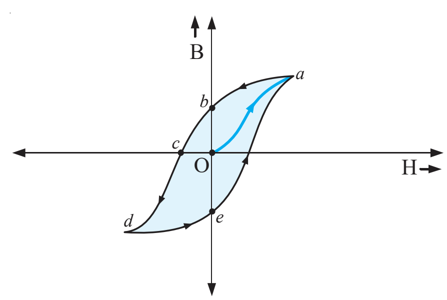
(vi) Soft ferromagnets : In some ferromagnetic materials, the magnetisation disappears on removal of the external field. Such materials are called soft ferromagnetic materials or soft ferromagnets.

Example **:** Nickel, Cobalt, Gadolinium.

Hysteresis :

It is the phenomeno of lagging behind of intensity of magnetization from the magnetizing field during the process of magnetization and demagnetization of a ferromagnetic material.

- Area within the $B - H$ loop represents the energy dissipated per unit volume within the material when it is carried through the cycle of magnetisation.
- **Residual Magnetism or Retentivity :** Magnetic induction left behind in the sample after magnetizing field has been removed is called retentivity. (Value of B at $H = 0$)
- **Coercivity :** The value of reverse magnetizing field required for residual magnetism of the sample to become zero. (Value of H at $B = O$)



The magnetic hysteresis loop is the B - H curve for ferromagnetic materials

Permanent magnets	Electromagnets
Made of ferromagnetic substances which retain magnetism for a long time at room temperature.	Soft iron has large permeability and small retentivity and hence is suitable for making electromagnets.
Methods to make permanent magnets. **(i)** Continuously run one end of a magnet on a fixed steel rod always in one direction. **(ii)** Pass a current through a solenoid containing a steel rod.	When a current is passed through a solenoid wound around a rod of soft iron, magnetic field inside the iron rod increases many times making it an electromagnet. On switching off the current, magnetic field more or less vanishes.
Used in compass, to make bar magnets etc.	Used in Electric bells, loud-speakers, in cranes to lift heavy things made of iron, etc.

Selection of Materials on the Basis of Hysteresis Curve :

1. For permanent Magnets :

Material should have *high retentivity* so that the magnet is strong and *high coercivity* so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations etc. Also the material should have *high permeability.* Steel is favoured. It has slightly smaller retentivity than soft iron but this is outweighed by the much smaller coercivity of soft iron. Other materials used are alnico, cobalt steel etc.

2. Electromagnets :

Core of electromagnets are made of ferromagnetic materials which have high permeability and low retentivity.

So that when we switch off the solenoid current, the Magnetism is effectively switched off.

Soft iron is suitable for core of electromagnet.

3. For Transformer Cores and Telephone Diaphragms :

Hysteresis curve must be narrow (Energy dissipation and heating will consequently be small).

The material must have high resistivity to lower eddy current losses.

Illustration - 40 *The magnetic moment of a magnet of mass 75 gm is 9×10^{-7} A-m^2. If the density of the material of magnet is 7.5×10^3 kg m^{-3}, then find the intensity of magnetisation.*

SOLUTION :

$$I = \frac{M}{V}, \text{ where volume, } V = \frac{mass\,(m)}{density(\rho)}$$

$$= \frac{M \times \rho}{m} = \frac{9 \times 10^{-7} \times 7.5 \times 10^3}{75 \times 10^{-3}} = 0.09\,A/m$$

Illustration - 41 *The permeability of substance is 6.28×10^{-4} wb/A-m. Find its relative permebility and suscepibility?*

SOLUTION :

$$\mu_r = \frac{\mu}{\mu_0} = \frac{6.28 \times 10^{-4}}{4 \times 10^{-7}} = 500$$

$$\mu_r = 1 + \chi \qquad \therefore \qquad \chi = \mu_r - 1 = 500 - 1 = 499$$

Illustration - 42 *An iron bar of length 10 cm and diameter 2 cm is placed in a magnetic field of intensity 1000 Am^{-1} with its length parallel to the direction of the field. Determine the magnetic moment produced in the bar if permeability of its material is 6.3×10^{-4} TmA^{-1}.*

SOLUTION :

We know that, $\mu = \mu_0(1 + \chi) \quad \Rightarrow \quad \chi = \frac{\mu}{\mu_0} - 1 = \frac{6.3 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 500.6$

Intensity of magnetisation, $I = \chi H = 500.6 \times 1000 = 5 \times 10^5 A^{-1}$

$\therefore$ Magnetic moment, $M = I \times V = I \times \pi r^2 l = 5 \times 10^5 \times 3.14 \times (10^{-2}) \times (10 \times 10^{-2}) = 17.70\ A - m^2$

Illustration - 43 *A magnetising field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross section 0.2 cm^2. Calculate permeability and susceptibility of the bar.*

SOLUTION :

Magnetic induction, $B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2\ Wb/m^2$

(i) Permeability, $\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4}\ TA^{-1}m$

(ii) As $\mu = \mu_0(1+\chi)$ then

Susceptibility, $\chi = \frac{\mu}{\mu_0} - 1 = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 596.1$

Illustration - 44 *A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23} Am^2. The sample is placed under a homogeneous magnetic field of 0.64 T, and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K? (Assume Curie's Law is valid)*

SOLUTION :

Initially, total dipole moment

$= 0.15 \times 1.5 \times 10^{-23} \times 2.0 \times 10^{24}$

$= 4.5\ Am^2$

Use Curie's Law $m \propto B/T$ to get the final dipole moment

$= 4.5 \times (0.98/0.84) \times (4.2/2.8)$

$= 7.9\ Am^2$

Illustration - 45 *Answer the following questions:*

(A) *Why does a paramagnetic sample display greater magnetization (for the same magnetising field) when cooled?*

(B) *Why is diamagnetism, in contrast, almost independent of temperature?*

(C) *If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?*

(D) *Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?*

(E) *Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point). Why?*

(F) *Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetization of a ferromagnet?*

SOLUTION :

(A) The tendency to disrupt the alignment of dipoles (with the magnetising field) arising from random thermal motion is reduced at lower temperatures.

(B) The induced dipole moment in a diamagnetic sample is always opposite to the magnetising field, no matter what the internal motion of the atoms is.

(C) Slightly less, since bismuth is diamagnetic.

(D) No, it is evident from the magnetisation curve. From the slope of magnetisation curve, it is clear that m is greater for lower fields.

(E) Proof of this important fact (of much practical use) is based on boundary conditions of magnetic fields (B and H) at the interface of two media. (When one of the media has μ >> 1, the field lines meet this medium nearly normally.) Details are beyond the scope of our curriculum.

(F) Yes. Apart from minor differences in strength of the individual atomic dipoles of two different materials, a paramagnetic sample with saturated magnetisation will have the same order of magnetisation. But of course, saturation requires impractically high magnetising fields.

IN-CHAPTER EXERCISE-C

1. Two identical magnetic dipoles of magnetic moment 2 Am^2 are placed at a separation of 2m with their axis perpendicular to each other in air. The resultant magnetic field at a mid-point between the dipoles is:

(A) $4\sqrt{5}\times10^{-5}T$ **(B)** $2\sqrt{5}\times10^{-5}T$

(C) $4\sqrt{5}\times10^{-7}T$ **(D)** $2\sqrt{5}\times10^{-7}$

2. A short bar magnet experiences a torque of magnitude 0.64 Nm, when it is placed in a uniform magnetic field of 0.32T, making an angle of 30° with the direction of the field. The magnetic moment of the magnet is:

(A) 1 Am^2 **(B)** 4 Am^2

(C) 6 Am^2 **(D)** None of these

3. A magnetised wire of magnetic moment M is bent in the form of a semicircle. The new magnetic moment is:

(A) M **(B)** $2M/\pi$

(C) M/π **(D)** None of the above

4. Two magnets of moment M and 2M are tied at an angle of 60° to each other. The magnetic moment of the combination will be:

(A) $\sqrt{5}M$ **(B)** $\sqrt{7}M$ **(C)** M **(D)** $2M$

5. The magnetic susceptibility of a material of a rod is 499. Permeability of vacuum is $4\pi\times10^{-7}$ Henry/m. Absolute permeability of the material of the rod in Henry/m is :

(A) $\pi\times10^{-4}$ **(B)** $2\pi\times10^{-4}$

(C) $3\pi\times10^{-4}$ **(D)** $4\pi\times10^{-4}$

6. A rigid circular loop of radius r and mass m lies in the x-y plane of a flat table and has a current i flowing in it. At this particular place the earth's magnetic field is $\vec{B}=B_x\hat{i}+B_z\hat{k}$. The value of i so that the loop start tilting is:

(A) $\dfrac{mg}{\pi r\sqrt{B_x^2+B_z^2}}$ **(B)** $\dfrac{mg}{\pi rB_x}$

(C) $\dfrac{mg}{\pi rB_z}$ **(D)** $\dfrac{mg}{\pi r\sqrt{B_x\,.\,B_z}}$

7. If a magnetic dipole of dipole moment M in stable equilibrium position is rotated through an angle θ with respect to the direction of the field H, then the work done is:

(A) $MH\sin\theta$ **(B)** $MH(1-\sin\theta)$

(C) $MH\cos\theta$ **(D)** $MH(1-\cos\theta)$

8. When a bar magnet is plaed at 90° to a uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to be half of the maximum value, at what angle should the magnet be inclined to the magnetic field?

(A) 60° **(B)** 30°

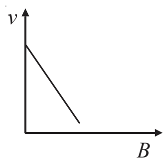
(C) 45° **(D)** 180°

9. A current carrying loop is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop be H. If a magnet is suspended inside the loop, it makes angle θ with H, then θ is equal to:

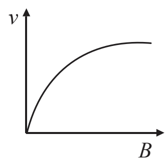
(A) $\tan^{-1}\left(\dfrac{H_0}{H}\right)$ **(B)** $\tan^{-1}\left(\dfrac{H}{H_0}\right)$

(C) $\text{cosec}^{-1}\left(\dfrac{H}{H_0}\right)$ **(D)** $\cot^{-1}\left(\dfrac{H_0}{H}\right)$

10. If frequency of oscillation of a magnet is plotted with strength of the field it oscillates in, the graph will be :



(A)



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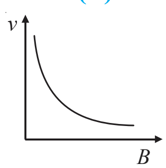
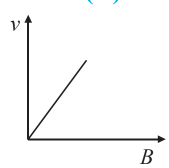


(C)



(D)

11. Two short magnets AB and CD are in the X-Y plane and parallel to X-axis and co-ordinates of their centres respectively are (0, 2) and (2, 0). Line joining the north-south poles of CD is opposite to that of AB and lie along the positive X-axis. The resultant field induction due to AB and CD at a point P(2, 2) is 100×10^{-2} T. When the poles of the magnet CD are reversed. The resultant field induction is 100×10^{-7} T. The value of magnetic moments of AB and CD (in Am^2) are :

(A) 300, 200 **(B)** 600,7400
(C) 200, 100 **(D)** 300, 150

12. Let B_V and B_H be the vertical and horizontal components of earth's magnetic fields at any points on earth's surface. Near the north pole :

(A) $B_v >> B_H$ **(B)** $B_v << B_H$
(C) $B_v = B_H$ **(D)** $B_v = B_H = 0$

13. The magnetic susceptibility of a paramagnetic substance at –73°C is 0.0060, then its value at –173°C will be :

(A) 0.0030 **(B)** 0.0120
(C) 0.0180 **(D)** 0.0045

14. Ferromagnetic materials used in a transformer must have :

(A) Low permeability and high hysteresis loss
(B) High permeability and Low hysteresis loss
(C) High permeability and high hysteresis loss
(D) Low permeability and Low hysteresis loss

15. A ship has to reach a place which is due west from its present position. Declination at this point is 15°. The direction from compass needle in which it should sail is :

(A) 75° **(B)** 15°
(C) 105° **(D)** 165°

16. At magnetic north pole of the earth, the value of horizontal component H and angle of dip θ is:

(A) $H = 0, \theta = 45°$ **(B)** $H \neq 0, \theta = 0°$
(C) $H = 0, \theta = 90°$ **(D)** $H \neq 0, \theta = 90°$

17. The magnetic field due to a short bar magnet of magnetic dipole moment M and length $2l$, on the axis at a distance z (where z >> l) from the centre of the magnet is given by formula :

(A) $\frac{\mu_0 M}{4\pi z^3}\hat{M}$ **(B)** $\frac{2\mu_0 M}{4\pi z^3}\hat{M}$
(C) $\frac{4\pi M}{\mu_0 z^2}\hat{M}$ **(D)** $\frac{\mu_0 M}{2\pi z^3}\hat{M}$

18. **Statement 1 :** Flux of magnetic field **B** through a closed surface is equal to zero.

$$\oint \vec{B}.\vec{ds} = 0$$

Statement 2 : Magnetic field lines are closed curves, they don't have any beginning or end.

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

19. The ratio of time periods of oscillation of two magnets in the same field is 2 : 1. If magnetic moment of both the magnets is equal, the ratio of the their moment of inertias will be :

(A) 4 : 1 **(B)** 1 : 4
(C) $\sqrt{2}$:1 **(D)** 1 : $\sqrt{2}$

20. A magnetic field of strength (H) $3 \times 10^3\ Am^{-1}$ produces a magnetic field of induction (B) of $12\pi T$ in an iron rod. Find the relative permeability of iron ?

(A) $4\pi \times 10^{-3}$ **(B)** 10^4
(C) 10^4 **(D)** $4\pi \times 10^{-7}$